

MDM4U Course Review

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. The number of patients treated in a dental office on Mondays was recorded for 11 weeks. What are the mean, median, and mode for this set of data?

5, 17, 28, 28, 28, 15, 13, 18, 10, 16, 20

- a. mean 17, median 18, mode 28
b. mean 18, median 17, mode 28
c. mean 16.5, median 18, mode 28
d. mean 28, median 17, mode 18

2. The final score of a hockey game is 4 to 3. How many different scores could there have been at the end of the first period?

- a. 12
b. 20
c. 7
d. 5

3. John owns a small Internet company and offers his clients three different formats for the name part of their e-mail address and seven different domains for the second part of the address. In how many ways can a person set up an e-mail address through John's company?

- a. 10
b. 21
c. 4
d. 12

Short Answer

4. The mean of the values 9, 11, 13, 21, 24, 18, and d is 17. Find d .
5. Each child in a study of infantile autism was given a behavioural test and graded on a scale from 0 (no symptoms) to 116 (maximum severity). The scores of the 21 children in the study were as follows.

27, 35, 65, 67, 47, 46, 63, 44, 34, 51, 17, 40, 41, 60, 24, 48, 29, 73, 60, 41, 47

Calculate the mean score, the standard deviation, and the variance.

A consumer magazine evaluated 39 models of bathroom scales. The table below lists the prices for these models (rounded to the nearest dollar).

Scale Model	Price (\$)	Scale Model	Price (\$)
EconoHealth A10	50	Superskale 6400	65
EconoHealth A12	50	Superskale 7200	20
EconoHealth B10	50	Superskale 8000	14
EconoHealth E10	28	Superskale 8280	25
EconoHealth Digital-10	65	SvelteChek 12300	24
EconoHealth E-20	40	SvelteChek 12400D	48

EconoHealth E-30	50	SvelteChek 12509	15
HealthSkale 190	22	SvelteChek 12510	10
HealthSkale 210	32	SvelteChek Fashion	17
HealthSkale 211	30	SvelteChek Pro	50
HealthSkale 290 Deluxe	79	SvelteChek Xtra	25
HealthSkale 310	50	Weighbeter 550	22
HealthSkale 1000	23	Weighbeter 801D	60
HealthSkale 1002	20	Weighbeter 830	30
HydroXact 12573	35	Weighbeter 835	30
HydroXact 12756	24	Weighbeter 950	10
HydroXact 12856	25	Weighbeter 2000	12
Prowt P10A	120	Weighbeter 2100	20
Prowt Value	35	Weighbeter Basic	12
Prowt Value 2	35		

6. Find the median, first quartile, and third quartile for the prices of these bathroom scales.

7. What is the z -score for the price of
 - a) the Weighbeter 801D scale?
 - b) the Weighbeter 830 scale?

8. Find the range and the interquartile range for the prices of these bathroom scales.

9. Bruna lives in Hamilton and is planning a trip to Hong Kong. On the day she wants to travel, she can take one of two flights to Toronto, then one of three possible flights to Vancouver, and finally one of four flights available from Vancouver to Hong Kong. Use a tree diagram to determine the number of ways Bruna could fly from Hamilton to Hong Kong.

10. In how many ways can a student answer all of the questions on a true/false test that has eight questions? Explain your solution.

11. In how many ways can the interviewers select a first, second, and third choice from a group of seven applicants for a position at a law firm?

In Switzerland the postal codes consist of two letters followed by four numbers.

12. How many postal codes are possible in the Swiss system if there are no restrictions on the choice of letters or numbers?

13. How many postal codes are possible in the Swiss system if you cannot use the letter O or the number 0?

14. How many ten-digit telephone numbers are possible if the first three digits must all be different?
15. A high-school hockey coach wants to try out a different goalie for each of the three periods of a practice game. In how many ways can the coach choose the goalies if there are nine students who want to play goal?
16. Which row of Pascal's triangle has terms that sum to 1024?
17. How is the sum of the entries in row 5 of Pascal's triangle related to powers of 2?
18. If 28% of the population of Statsville wears contact lenses, 37% have blue eyes, and 9% are blue-eyed people who wear contact lenses, what is the probability that a randomly selected resident has neither blue eyes nor contact lenses?

Problem

19. The following table summarizes data collected in a survey of owners of small trucks. Most owners reported their distances rounded to the nearest 100 km.

Distance Travelled Annually (km)	Number of Trucks
5 000–6 999	5
7 000–8 999	10
9 000–10 999	12
11 000–12 999	20
13 000–14 999	20
15 000–16 999	14
17 000–18 999	11
19 000–21 000	4

Estimate the mean distance these trucks were driven annually.

20. How many arrangements of five letters from the word *certain* do not contain the letter *t*? Explain your reasoning.
21. How many arrangements of five letters from the word *certain* contain the letter *t*? Explain your reasoning.
22. You are setting up a system of programming codes for a universal TV remote. Each code will have five digits and cannot include a 0 or repeat any digits. How many codes
 - a) do not contain a 7?
 - b) have 3 as the third digit and 5 as the fifth digit?
 - c) begin with an even number?
 - d) begin and end with an even number?

23. You work as a health inspector and must visit each of the 15 restaurants in town once each week.
- In how many different orders can you make these inspections?
 - If you were to work 50 weeks a year and use a different order every week, how long would it take you to try all of the different possible orders?
24. Gisela lives in a subdivision where the streets are laid out in a grid pattern. Her school is eight blocks south and six blocks east of her home. How many different direct routes could she take when she bicycles to school if she always stops by her friend's house, which is three blocks south and four blocks east of Gisela's. Explain your reasoning.
25. Gord and Tien are skateboard enthusiasts and are trying out different routes between their homes. Gord lives four blocks north and six blocks west of Tien's home.
- How many different routes can Gord and Tien try without going out of their way?
 - How is your answer related to Pascal's triangle?
26. A fabric store has 44 bolts of fabric on its clearance table. Of these bolts, 28 are broadcloth, 17 are floral prints, 20 have a blue background, 12 are floral prints on broadcloth, 10 are broadcloth with a blue background, and 4 are floral prints with a blue background. If all but 2 of the clearance fabrics fit at least one of these categories, how many of the bolts are broadcloth floral prints with a blue background? Illustrate your answer with a Venn diagram.
27. There are 10 councillors and 12 planning department staff available to serve on a budget committee for the new city council. If the committee will consist of 3 councillors and either 1 or 2 planning staff, how many different committees could the council choose?
28. A 15-passenger shuttle bus takes athletes between venues at the Canada Summer Games. If 32 athletes need to get to the track and field stadium, in how many ways can passengers be chosen for
- the bus's first trip?
 - the bus's second trip?
29. The students producing a school fashion show plan to have five scenes with music between them. The music students have come up with 18 pieces: 6 for piano, 5 for recorder, and 7 for guitar. The students want to use at least 1 piece for the piano. In how many ways can the group choose the 4 pieces of bridging music? Explain your reasoning.
30. Tom is practising archery with a target has three concentric zones: a circular bull's-eye in the centre, an inner ring, and an outer ring. He has an 0.12 probability of hitting the bull's-eye, an 0.37 probability of hitting the inner ring, and an 0.43 probability of hitting the outer ring. On an given shot, what is the probability that Tom
- misses the target?
 - hits the target but does not get a bull's-eye?
 - hits the inner ring or the bull's-eye?

31. If you were to toss four coins, what are the odds in favour of at least two landing heads up?
32. A six-member working group to plan a student common room is to be selected from five teachers and nine students. If the working group is randomly selected, what is the probability that it will include at least two teachers?
33. A survey at a school asked students if they were ill with a cold or the flu during the last month. The results were as follows. None of the students had both a cold and the flu.

	Cold	Flu	Healthy
Females	32	18	47
Males	25	19	38

Use these results to estimate the probability that

- a randomly selected student had a cold in the last month
 - a randomly selected female student was healthy last month
 - a randomly selected student who had the flu last month is male
 - a randomly selected male student had either a cold or the flu last month
34. A target for a dart game has three concentric circles with radii of 6 cm, 10 cm, and 18 cm. If you hit this target by throwing a dart randomly, what are the probabilities of landing in each of the three regions?
35. Ninety percent of a country's population are right-handed.
- What is the probability that exactly 29 people in a group of 30 are right-handed?
 - What is the expected number of right-handed people in a group of 30?
 - Design a simulation to show that the expectation calculated in part b) is accurate.
36. A manufacturer of cereal finds that the masses of cereal in the company's 200-g packages are normally distributed with a mean of 200 g and a standard deviation of 16.3 g.
- What proportion of these boxes contain between 183.7 g and 216.3 g of cereal?
 - What is the probability that a box selected at random contains more than 216.3 g of cereal?
 - How many of a shipment of 120 boxes would you expect to contain between 178 g and 225 g of cereal?
 - Determine the range of masses that you would expect 90% of these boxes of cereal to contain.

37. It is estimated that 10% of the vehicles entering Canada from the United States carry undeclared goods. Use the normal approximation to calculate the probability that a search of 500 randomly selected vehicles will find fewer than 50 with undeclared goods.

MDM4U Course Review
Answer Section

MULTIPLE CHOICE

1. ANS: B PTS: 1
2. ANS: B PTS: 1
3. ANS: B PTS: 1

SHORT ANSWER

4. ANS:
23

PTS: 1

5. ANS:
Since the study is trying to determine the characteristics of the population of all autistic children, use the formulas for calculating statistics for a sample.
 $\bar{x} = 45.7, s = 15.1, s^2 = 229$

PTS: 1

6. ANS:
median \$30, first quartile \$20, third quartile \$50

PTS: 1

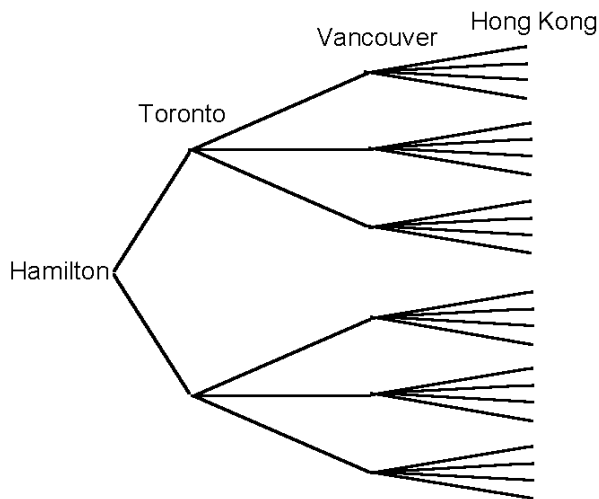
7. ANS:
a) 1.13
b) -0.24

PTS: 1

8. ANS:
range \$110, interquartile range \$30

PTS: 1

9. ANS:



There are 24 ways Bruna could choose her flights.

PTS: 1

10. ANS:

For each question there are two choices. Applying the multiplicative (fundamental) counting principle, there are $2^8 = 256$ ways a student could answer the test.

PTS: 1

11. ANS:

There are 210 ways to select the top three choices.

PTS: 1

12. ANS:

There are 6 760 000 postal codes possible.

PTS: 1

13. ANS:

There are 4 100 625 postal codes possible.

PTS: 1

14. ANS:

There are 7 200 000 000 telephone numbers possible.

PTS: 1

15. ANS:

There are 504 ways to choose the goalies.

PTS: 1

16. ANS:

row 10

PTS: 1

17. ANS:

The sum of the entries in row 5 is $32 = 2^5$.

PTS: 1

18. ANS:

44%

PTS: 1

PROBLEM

19. ANS:

$$\begin{aligned}\mu &= \frac{\sum_i f_i m_i}{\sum_i f_i} \\ &= \frac{5(6000) + 10(8000) + 12(10\,000) + 20(12\,000) + 20(14\,000) + 14(16\,000) + 11(18\,000) + 4(20\,000)}{5 + 10 + 12 + 20 + 20 + 14 + 11 + 4} \\ &= \frac{1\,252\,000}{96} \\ &= 13\,042\end{aligned}$$

These trucks were driven about 13 000 km a year.

PTS: 1

20. ANS:

For the first letter, there are six choices. For the second letter, there are five choices left; for the third, four; for the fourth, three; and for the fifth, two. Applying the multiplicative counting principle, the total number of arrangements is $6 \times 5 \times 4 \times 3 \times 2 = 720$.

PTS: 1

21. ANS:

Direct method

The letter t can be any of the five letters in the arrangement. If t is the first letter, there are six choices left for the second letter, five choices for the third letter, four for the fourth, and three for the fifth. Using the multiplicative counting principle, there are $6 \times 5 \times 4 \times 3 = 360$ arrangements with t as the first letter. There are the same number of arrangements with the letter t in each of the other four positions. Thus, the total number of five-letter arrangements that include the letter t is $5 \times 360 = 1800$.

Indirect method

Find the total number of five-letter arrangements and subtract those that do not contain the letter t . The total number of five-letter arrangements is $7 \times 6 \times 5 \times 4 \times 3 = 2520$. In arrangements without the letter t , there are six choices for the first letter. For the second letter, there are five choices left; for the third, four; for the fourth, three; and for the fifth, two. Applying the multiplicative counting principle, the number of arrangements without the letter t is $6 \times 5 \times 4 \times 3 \times 2 = 720$. Therefore, the number of arrangements with the letter t is $2520 - 720 = 1800$.

PTS: 1

22. ANS:

- a) No digit can be either 0 or 7, so there are eight possibilities for the first digit, seven for the second, six for the third, and so on. The number of possible codes would be $8 \times 7 \times 6 \times 5 \times 4 = 6720$.
- b) Since 3 and 5 are already used, there are seven possibilities for the first digit, six for the second digit, and five for the fourth digit. The number of possible codes would be $7 \times 6 \times 5 = 210$.
- c) The first digit can be 2, 4, 6, or 8. There are eight choices left for the second digit, seven for the third digit, and so on. The number of possible codes would be $4 \times 8 \times 7 \times 6 \times 5 = 6720$.
- d) Since both the first and last digit must be even, there are four choices for the first digit and three for the last digit. There are eight choices left for the second digit, seven for the third digit, and six for the fourth digit. The number of possible codes would be $4 \times 3 \times 8 \times 7 \times 6 = 4032$.

PTS: 1

23. ANS:

a) ${}_{15}P_{15} = 15!$

$= 1.307\ 674\ 368 \times 10^{12}$

b) It would take $(1.307\ 674\ 368 \times 10^{12}) \div 50 = 2.615\ 348\ 74 \times 10^{10}$ years.

PTS: 1

24. ANS:

Consider the two parts of Gisela's route separately. The number of direct routes Gisela can take from her home to her friend's house is equal to the number of different orders in which she can choose to travel the three blocks east and the four blocks south. Therefore, the formula for permutations with some identical items can be applied. The

$$\frac{7!}{3!4!} = 35$$

number of routes between the two houses is $\frac{7!}{3!4!} = 35$. Similarly, the number of routes from the friend's house to

$$\frac{7!}{5!2!} = 21$$

the school is $\frac{7!}{5!2!} = 21$. Using the multiplicative counting principle, the different routes from Gisela's home to her school via her friend's house is $35 \times 21 = 735$.

PTS: 1

25. ANS:

a) The number of possible routes is 210, as shown below.

Gord's Home		1	1	1	1	1	1
1		2	3	4	5	6	7
1		3	6	10	15	21	28
1		4	10	20	35	56	84
1		5	15	35	70	126	210
							Tien's Home

b) The number of possible routes is equal to the term $t_{10,4}$ in Pascal's triangle.

PTS: 1

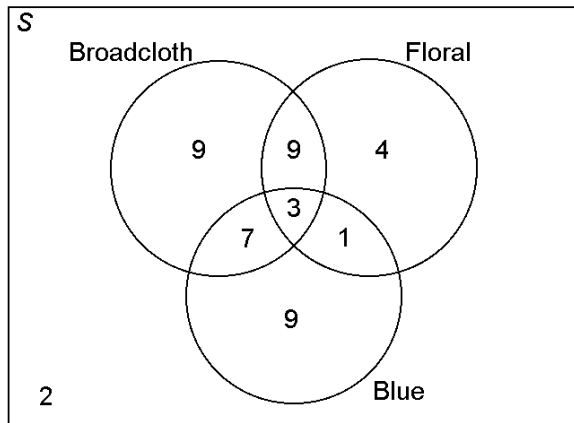
26. ANS:

Let n be the number of bolts of broadcloth floral prints with a blue background. Applying the principle of inclusion and exclusion,

$$44 - 2 = 28 + 17 + 20 - 12 - 10 - 4 + n$$

$$n = 3$$

There are 3 bolts of broadcloth floral prints with a blue background.



PTS: 1

27. ANS:

$$C(10, 3) \times C(12, 1) + C(10, 3) \times C(12, 2) = 120(12) + 120(66) \\ = 9360$$

There are 9360 different committees the council could choose.

PTS: 1

28. ANS:

a) $C(32, 15) = 565\,722\,720$

b) $C(17, 15) = 136$

PTS: 1

29. ANS:

Direct Method

There are 6 pieces for piano and 12 for the other instruments. The students can choose 1, 2, 3, or 4 piano pieces. Consider each of these cases in turn.

1 piano piece: The students can choose the piano piece in $C(6, 1)$ ways and the remaining 3 pieces in $C(12, 3)$ ways. The number of combinations with 1 piano piece is $C(6, 1) \times C(12, 3) = 1320$.

2 piano pieces: The number of combinations is $C(6, 2) \times C(12, 2) = 990$.

3 piano pieces: The number of combinations is $C(6, 3) \times C(12, 1) = 240$.

4 piano pieces: The number of combinations is $C(6, 4) \times C(12, 0) = 15$.

The number of combinations that include at least 1 piano piece is the total of these four cases, 2565.

Indirect Method

Find the total number of possible combinations and subtract those that do not have any piano pieces:

$$C(18, 4) - C(12, 4) = 3060 - 495 \\ = 2565$$

PTS: 1

30. ANS:

$$P(\text{miss}) = 1 - (0.12 + 0.37 + 0.43)$$

a) $= 0.08$ or 8%

$$P(\text{inner or outer}) = 0.37 + 0.43$$

b) $= 0.80$ or 80%

$$P(\text{inner or bull's-eye}) = 0.37 + 0.12$$

c) $= 0.49$ or 49%

PTS: 1

31. ANS:

$$\begin{aligned} P(\text{tossing at least two heads}) &= 1 - P(\text{no heads}) - P(\text{one head}) \\ &= 1 - \frac{1}{16} - \frac{4}{16} \\ &= \frac{11}{16} \end{aligned}$$

Therefore, the odds in favour of tossing at least two heads are 11:5.

PTS: 1

32. ANS:

$$P(\text{at least two teachers}) = 1 - P(\text{no teachers}) - P(\text{one teacher})$$

$$\begin{aligned} &= 1 - \frac{\binom{9}{6} \binom{5}{0}}{\binom{14}{6}} - \frac{\binom{9}{5} \binom{5}{1}}{\binom{14}{6}} \\ &= \frac{3003 - 84 - 630}{3003} \\ &= \frac{2289}{3003} \text{ or about } 0.7622 \end{aligned}$$

PTS: 1

33. ANS:

$$P(\text{cold}) = \frac{57}{179}$$

a) $\doteq 0.3184$ or about 31.8%

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} :$$

b) Using the conditional probability formula,

$$\begin{aligned}
 P(\text{healthy}|\text{female}) &= \frac{\frac{47}{179}}{\frac{179}{97}} \\
 &= \frac{47}{97} \\
 &\doteq 0.4845 \text{ or about } 48.5\%
 \end{aligned}$$

$$\begin{aligned}
 P(\text{male}|\text{flu}) &= \frac{19}{37} \\
 &\doteq 0.5135 \text{ or about } 51.3\%
 \end{aligned}$$

- c) Restricting the sample space to only those who had the flu,
 d) Restricting the sample space to only males,

$$\begin{aligned}
 P(\text{cold or flu}|\text{male}) &= \frac{44}{82} \\
 &\doteq 0.5366 \text{ or about } 53.7\%
 \end{aligned}$$

PTS: 1

34. ANS:

$$\begin{aligned}
 \text{Area of inner region} &= \pi(6)^2 \\
 &= 36\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of middle region} &= \pi(10)^2 - \pi(6)^2 \\
 &= 64\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of outer region} &= \pi(18)^2 - \pi(10)^2 \\
 &= 224\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Total area of target} &= \pi(18)^2 \\
 &= 324\pi \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 P(\text{inner region}) &= \frac{36\pi}{324\pi} \\
 &= \frac{1}{9} \text{ or about } 0.11
 \end{aligned}$$

$$\begin{aligned}
 P(\text{middle region}) &= \frac{64\pi}{324\pi} \\
 &= \frac{16}{81} \text{ or about } 0.20
 \end{aligned}$$

$$\begin{aligned}
 P(\text{outer region}) &= \frac{224\pi}{324\pi} \\
 &= \frac{56}{81} \text{ or about } 0.69
 \end{aligned}$$

PTS: 1

35. ANS:

- a) Here, $p = 0.90$ and $n = 30$.

$$\begin{aligned}
 P(29) &= {}_{30}C_{29}(0.90)^{29}(0.10)^1 \\
 &= 0.14
 \end{aligned}$$

This probability can also be calculated using the **binompdf**(function on a graphing calculator, the **BINOM-DIST** function in a spreadsheet, or the **binomialProbability** function in Fathom™.

b) $np = 30 \times 0.90$
 $= 27$

c) Answers may vary. The following method could be used with a graphing calculator.

Check that list L1 is clear.

Let 1 represent a right-handed person.

Enter `randBin(1,9/10,30)`→L1.

Enter `sum(L1)`.

Record this result.

Use this process to generate ten or more sets of random numbers and calculate the average of the sums.

PTS: 1

36. ANS:

The probabilities can be calculated on a graphing calculator with the **normalcdf**(function, as shown below. They can also be found either by using the **NORMDIST** function in a spreadsheet or by converting to z -scores and using the table Areas Under the Normal Distribution Curve on page 606 of the student textbook.

a) Since $P(183.7 < X < 216.3) = 0.683$, about 68.3% of the boxes will contain between 183.7 g and 216.3 g of cereal.

```
normalcdf(183.7,
216.3,200,16.3)
.6826894809
normalcdf(216.3,
1E99,200,16.3)
.1586552596
```

b) From the second calculation in the screen above, $P(X > 216.3) = 0.159$.

c)

```
normalcdf(178,22
5,200,16.3)
.8488961955
Ans*120
101.8675435
```

The probability that a box will have between 178 g and 225 g of cereal is $P(178 < X < 225) = 0.849$. The expected number of boxes containing this range of masses is $120 \times 0.849 = 101.9$.

d) Since a normal distribution is symmetric about the mean, the range of masses will be centred on the mean mass of 200 g and $P(-z < Z < z) = 0.90$, where z is the z -score of the upper limit of the range of masses. Thus, $P(Z > -z)$ must be 0.05. In the table of Areas Under the Normal Distribution Curve, $P(Z > -z) = 0.05$ for $z = 1.645$. Therefore, 90% of the boxes will contain masses of cereal within 1.645 standard deviations of the mean.

$1.645 \times 16.3 \text{ g} = 26.8 \text{ g}$

So, the corresponding range of masses is $200 \text{ g} \pm 26.8 \text{ g}$, or 173.2 g to 226.8 g.

PTS: 1

37. ANS:

A graphing-calculator solution is shown below. The probability can also be calculated by using the **NORMDIST** function in a spreadsheet or by converting to z-scores and using the table of Areas Under the Normal Distribution Curve on page 606 of the student textbook.

Here, $n = 500$, $p = 0.10$, and $q = 0.90$. To find the probability of fewer than 50 vehicles with undeclared goods, use a continuity correction with 49.5 as the upper limit.

```
normalcdf(-1E99,
49.5,500*0.1,(50
0*0.1*0.9)^.5)
.4702920381
```

The probability of less than 50 vehicles with undeclared goods is about 47%.

PTS: 1